Resonant Cooper Pair Tunneling: Quantum Noise and Measurement Characteristics

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We study the quantum charge noise and measurement properties of the *double* Cooper pair resonance point in a superconducting single-electron transistor (SSET) coupled to a Josephson charge qubit. Using a density matrix approach for the coupled system, we obtain a full description of the measurement back-action; for weak coupling, this is used to extract the quantum charge noise. Unlike the case of a non-superconducting SET, the back-action here can induce population inversion in the qubit. We find that the Cooper pair resonance process allows for a much better measurement than a similar non-superconducting SET, and can approach the quantum limit of efficiency.

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Among the many open issues related to solid state quantum computation, the question of how best to *measure* a solid state qubit remains a particularly interesting one. In the case where the qubit is a Cooper pair box (i.e. a Josephson-junction single charge box), the standard choice for a read-out device is the single-electron transistor (SET) [1–6]. An alternate and potentially more powerful approach is to use a *superconducting* single electron transistor (SSET) biased at a point where the cyclic resonant tunneling of Cooper pairs dominates transport [7–11] [12]. Such processes, known as Josephson quasiparticle (JQP) resonances, would appear to be an attractive choice for use in a measurement as their resonance structure implies an extremely high sensitivity. However, precisely because of their large gain, these processes may be expected to strongly alter the state of the qubit in a measurement. To assess the balance between these two opposing tendencies, a close examination of the physics of JQP tunneling is required. Note that the SSET-qubit system is more relevant to experiment than a setup having a SET, as fabrication typically results in the entire system being made from a single metal.

In this paper, we focus on a *double* JQP process (DJQP) (see Fig. 1), which occurs at a lower SSET source-drain voltage than single JQP processes, and which has been used in a recent experiment [13]. We assess the potential of DJQP to act as a one-shot measurement of the state of a Cooper pair box qubit. This involves characterizing both $\tau_{\rm meas}$, the time needed to discriminate the two qubit states in the measurement, and the back-action of the measurement on the qubit, which is described by a mixing rate $\Gamma_{\rm mix}$ and a dephasing rate τ_φ. These quantities are intimately related to the noise properties of the SSET, which are of fundamental interest in themselves, given the novel nature of the DJQP process. $\tau_{\rm meas}$ is determined largely by the shot noise of the process, while $\Gamma_{\rm mix}$ and τ_φ are related to the associated charge noise on the SSET island. While the shot noise of a *single* JQP process has been analyzed recently [14], the quantum charge noise has not been addressed. It is of particular interest, as the experiment of

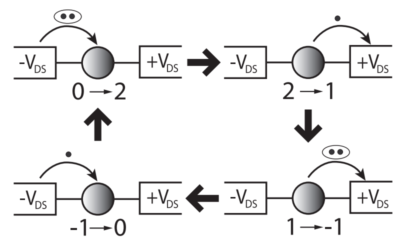

FIG. 1. Schematic showing the four steps of the double Josephson quasiparticle process which can occur in a superconducting single-electron transistor. Circles represent the central island of the SSET, while the rectangles are the electrodes. Numbers indicate the charge of the SSET island.

Ref. [13] uses the ability of a qubit to act as a spectrum analyzer of this quantum charge noise.

To describe the measurement process in our system, we employ a density matrix description of the *fully coupled* SSET plus qubit system; this is similar to the approach taken by Makhlin *et al.* [4] for a SET, but extended to deal with Josephson tunneling. This approach is not limited by a requirement of weak-coupling, as are standard approaches which perturbatively link $\Gamma_{\rm mix}$ to the transistor charge noise [5,6]; nonetheless, in the limit of weak-coupling the present method can be used to calculate the quantum charge noise of the SSET. We find that the quantum (i.e. asymmetric in frequency) nature of the noise is particularly pronounced for the DJQP feature, leading to regimes where the SSET can strongly relax the qubit. Moreover, due to the resonant nature of Cooper pair tunneling, there exist regimes where the SSET can cause a pronounced *population inversion* in the Cooper pair box, something that is impossible using a SET. For typical device parameters, we find that a far better single-shot measurement is possible using the DJQP process than with a comparable SET.

Model– The Hamiltonian of the coupled qubit plus SSET system is is written as $\mathcal{H} = \mathcal{H}_{\rm S} + \mathcal{H}_{\rm Q} + \mathcal{H}_{\rm int}$. The qubit itself (or "box"), described by $\mathcal{H}_{\rm Q}$, consists of a superconducting metal island in the Coulomb blockade regime where only two charge states are relevant. These can be regarded as the σ_z eigenstates of a fictitious spin

1/2. The island is attached via a tunnel junction to a bulk superconducting electrode, leading to the form

$$\mathcal{H}_Q = -\frac{1}{2}\left[\left(4E_{CQ}(1-\mathcal{N}_Q)\right)\sigma_z + E_{JQ}\sigma_x\right]. \qquad (1)$$

where E_{CQ} is the charging energy of the box, E_{JQ} is the Josephson coupling energy of the box, and $\mathcal{N}_Q$ is the dimensionless gate voltage applied to the box. The SSET consists of a superconducting, Coulomb-blockaded island which is attached via tunnel junctions to two superconducting electrodes (Fig. 1). The SSET Hamiltonian $\mathcal{H}_S = H_K + H_C + H_V + H_T$ has a term H_K describing the kinetic energy of source, drain and central island electrons, a term H_V which describes the work done by the voltage sources, and a tunneling term H_T. The charging term is $H_C = E_{CS}(n_S - \mathcal{N}_S)^2$, where E_{CS} is the SSET charging energy, n_S is the number of electrons on the central island, and $\mathcal{N}_S$ is the dimensionless gate voltage applied to the island. Finally, the qubit is capacitively coupled to the SSET: $\mathcal{H}_{\rm int} = 2E_{CQ}\frac{C_C}{C_\Sigma}\sigma_z n_S \equiv E_{\rm int}\sigma_z n_S$. Here C_C is the cross-capacitance between the box and the central island of the SSET, and C_Σ is the total capacitance of the SSET island. Note that we neglect the coupling of the qubit to its environment, as we are interested here in the intrinsic effect of the SSET on the qubit [15]. We also assume a SSET with identical tunnel junctions, whose dimensionless conductance g satisfies $g/(2\pi) \ll 1$.

The DJQP process occurs when the SSET gate voltage $\mathcal{N}_S$ and drain-source voltage $2V_{DS}$ are such that two Cooper-pair tunneling transitions (one in each junction) are resonant. We label these transitions as $n_s = 0 \to 2$ (left junction) and $n_s = 1 \to -1$ (right junction) (see Fig. 1). Resonance thus requires $eV_{DS} = E_{CS}$ and $\mathcal{N}_S = 1/2$. In addition, E_{CS}/Δ_S (where Δ_S is the superconducting gap of the SSET) must be chosen so that the quasiparticle transitions linking the two Cooper pair resonances are energetically allowed (i.e. $n_S = 2 \to 1$ and $n_S = -1 \to 0$), whereas transitions which end the cycle (i.e. $n_S = 0 \to 1$) are not. We take $E_{CS} = \Delta_S$ to satisfy these conditions; this corresponds to the experiment of Ref. [13]. The two quasiparticle transitions which occur in the DJQP are characterized by a rate Γ, which is given by the usual expression for quasiparticle tunneling between two superconductors [16]. The effective Cooper pair tunneling rate γ_J emerging from our description (i.e. Eq. (3) below) is given by [8]:

$$\gamma_J(\delta) = \frac{E_{JS}^2\Gamma}{4\left(\delta^2 + (\Gamma/2)^2\right)} \qquad (2)$$

Here, δ is the energy difference between the two charge states involved in tunneling, E_{JS} is the Josephson energy of the SSET, and we set $\hbar = 1$.

Calculation Approach– We consider the reduced density matrix ρ of the qubit plus SSET system obtained by tracing out the SSET fermionic degrees of freedom. The evolution of ρ is calculated perturbatively in the tunneling Hamiltonian H_T, keeping only the lowest order terms; this corresponds to the neglect of co-tunneling processes, which is valid for small g and near the DJQP resonance. Using an interaction representation where $\mathcal{H}_T$ is viewed as a perturbation, the equation of motion of ρ takes the standard form:

$$\frac{d}{dt}\rho(t) = -\int_{-\infty}^{t} dt' \langle[\mathcal{H}_T(t), [\mathcal{H}_T(t'), \rho(t')\otimes\rho_F]]\rangle \qquad (3)$$

The angular brackets denote the trace over SSET fermion degrees of freedom; as we work at zero temperature, ρ_F is the density matrix corresponding to ground state of these degrees of freedom in the absence of tunneling. In the diagrammatic language of Ref. [17], Eq. (3) is equivalent to keeping all $\mathcal{H}_T^2$ terms in the self-energy of the Keldysh propagator governing the evolution of ρ. Note that the correlators in Eq. (3) describe both quasiparticle tunneling and Josephson tunneling in the SSET.

To make further progress, we treat the Josephson coupling emerging from Eq. (3) as energy-independent and given by the Ambegaokar-Baratoff value $E_{JS} = g\Delta_S/8$. We also use the smallness of g to neglect logarithmic renormalization terms, as was done in Ref. [4]. One can then immediately solve for the time-independent solution of Eq. (3), which describes the quasi-equilibrium state achieved by the system after all mixing and dephasing of the qubit by the SSET has occurred. To describe the dynamics of mixing (i.e. the relaxation of the qubit state populations to their stationary value), we also calculate the corresponding eigenmode of Eq. (3). A Markov approximation is made which involves replacing $\rho(t')$ by $\rho(t)$ on the RHS of Eq. (3); for the mixing mode, this should be done in the Schrödinger picture [18]. This approximation is justified as long as the time dependence of ρ in the mixing mode is weak compared to typical frequencies appearing in the correlators of Eq. (3), requiring here that $\Gamma_{\rm mix} < \Gamma, E_{CS}$ and $E_{JS} < E_{CS}$ [18].

Back-Action– We focus here primarily on the mixing effect of the measurement back-action; dephasing will be discussed more extensively in Ref. [18]. The mixing rate $\Gamma_{\rm mix} = \Gamma_{\rm rel} + \Gamma_{\rm exc}$ is set by the rates at which the measurement relaxes and excites the qubit. Let Ω denote the $\mathcal{N}_Q$-dependent energy difference between the two qubit states. For weak coupling ($E_{\rm int} \ll \Omega$), Fermi's Golden rule relates $\Gamma_{\rm rel}$ and $\Gamma_{\rm exc}$ to the quantum charge noise of the SSET island $S_Q(\omega) = \int dt\, e^{-i\omega t}\langle n_S(t)n_S(0)\rangle$:

$$\Gamma_{\rm rel/exc} = E_{\rm int}^2\left(\frac{E_{JQ}}{\Omega}\right)^2 S_Q(\pm\Omega). \qquad (4)$$

In our approach, these rates may be directly obtained by using the stationary solution (which gives the post-mixing occupancies of the box eigenstates) and the mixing eigenvalue of Eq. (3). In the limit of weak coupling, one can then use Eq. (4) to extract $S_Q(\Omega)$. Our method

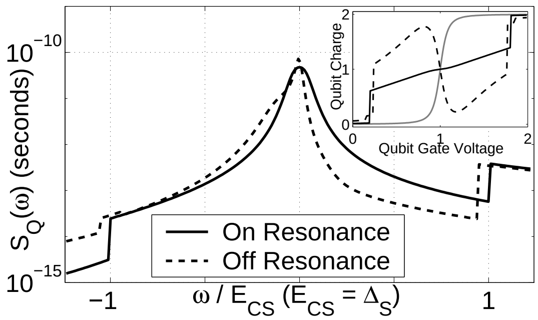

FIG. 2. Quantum charge noise associated with the DJQP process. The solid curve corresponds to $\mathcal{N}_S$,V_{DS} tuned to the center of the DJQP resonance; the dashed curve corresponds to moving eV_{DS} away from resonance by $+\Gamma/4$. We take $g = 0.5$ and $\Delta_S \simeq E_{CS} = 0.25meV$ in the SSET, corresponding to the device of Ref. 13; this gives $E_{JS}/(h\Gamma) \simeq 0.04$. Inset: average qubit charge after mixing has occurred for weak coupling ($E_{\rm int}/E_{JQ} = 0.01$), as a function of qubit gate voltage $\mathcal{N}_Q$; see text for details. As in Ref. 13, we have $E_{CQ} \simeq 77\mu eV$ and $E_{JQ} \simeq 27\mu eV$. The frequency range probed by tuning $\mathcal{N}_Q$ matches the range of the main plot; the sharp steps in the average charge occur at $\Omega(\mathcal{N}_Q) \simeq E_{CS}$.

for calculating the quantum noise, which uses the qubit as a spectrum analyzer, is physically intuitive and no more difficult to implement than standard approaches [6]; in addition, we are able to calculate $\Gamma_{\rm rel}$ and $\Gamma_{\rm exc}$ when the coupling is not weak, and Eq. (4) fails.

Fig. 2 displays the quantum charge noise obtained at zero temperature, using SSET parameters which correspond to Ref. [13]. The solid curve in Fig. 2 is for the center of the DJQP resonance– $\mathcal{N}_S = 1/2$, $eV_{DS} = E_{CS}$. Note the sudden asymmetry that develops between absorption (i.e. $S_Q(+|\omega|)$) and emission (i.e $S_Q(-|\omega|)$) when $|\omega|$ increases beyond E_{CS}. These sudden jumps correspond to the opening and closing of transport channels in the SSET, and their sharpness (which is absent for similar processes in a SET [6]) is a direct consequence of the singularity in the quasiparticle density of states. For example, as ω rises past E_{CS}, quasiparticle transitions which are normally forbidden in the DJQP cycle (i.e. $n_S = 0 \to 1$) suddenly become energetically allowed *if* they absorb energy from the qubit, causing a sudden increase in $S_Q(\omega)$.

The effect of the SSET quantum charge noise on the qubit is shown in the inset of Fig. 2, where the average qubit charge $\langle N_B \rangle \equiv 1 + \langle \sigma_z \rangle$ for $t \gg \tau_{mix}$ is shown as a function of $\mathcal{N}_Q$. Changing $\mathcal{N}_Q$ tunes the qubit splitting frequency Ω, allowing one to effectively probe the frequency dependence of the noise. The solid black curve corresponds to being at the center of the DJQP feature, while the grey curve corresponds to the unperturbed qubit ground state. The sharp features in the quantum noise manifest themselves in $\langle N_B \rangle$, a quantity which is directly accessible in experiment.

Even more interesting are features emerging in the low frequency quantum noise ($|\omega| \ll E_{CS}$) when one tunes $\mathcal{N}_S$ or V_{DS} slightly off the DJQP resonance center. Unlike the case of a SET, where asymmetries in the noise are weak for these frequencies, there are strong features here that result from the resonant nature of Cooper pair tunneling. By treating the mixing terms in Eq. (3) perturbatively, simple analytic expressions can be obtained for the quantum noise in this regime when $E_{JS} < \Gamma$ (in Ref. [13], $E_{JS}/(h\Gamma) \simeq 0.04$). If one moves away from the DJQP center by tuning only V_{DS} (i.e. $\mathcal{N}_S = 1/2$, $eV_{DS} = E_{CS} + \delta_V/2$), we find ($|\omega| < E_{CS}$):

$$S_Q(\omega) = \gamma_J(\delta_V)\frac{[\gamma_J(\delta_V+\omega)/\gamma_J(\delta_V-\omega)]}{[4\gamma_J(\delta_V+\omega)\gamma_J(\delta_V-\omega)]+\omega^2} \qquad (5)$$

In the limit where ω is much smaller than the width $\Gamma/2$ of the Cooper pair resonance, Eq. (5) simply corresponds to classical telegraph noise (the SSET only spends appreciable time in the states $n_S = 0$ and $n_S = 1$). However, for finite δ_V and ω, Eq. (5) indicates that the noise develops a pronounced asymmetry, even though $|\omega| \ll E_C$. In particular, if $\delta_V > 0$, one has $S_Q(-|\omega|) > S_Q(+|\omega|)$, implying that *emission by the SSET exceeds absorption*. This behavior is shown by the dashed curves in Fig. 2, which correspond to $\mathcal{N}_S = 1/2$, $\delta_V = +\Gamma/4$. This effect is a direct consequence of the resonant nature of Cooper pair tunneling– by emitting energy, *both* Cooper pair tunneling processes in the DJQP cycle become more resonant, while absorbing energy pushes them even further from resonance. The net result is a population inversion in the qubit at zero temperature, which in turn leads to a striking, non-monotonic dependence of qubit charge on $\mathcal{N}_Q$ (this is shown by the dashed curve in the inset of Fig. 2) [15]. Note that if one moves away from the center of the DJQP resonance by only changing the gate voltage $\mathcal{N}_S$, no asymmetry in the noise results, as now emission (or absorption) moves one of the Cooper-pair transitions in the DJQP process further *towards* resonance, while it moves the other transition further *away* from resonance. Nonetheless, the noise in this case still has a non-monotonic dependence on frequency. Letting $\delta_V = 0$ and $\delta_{\mathcal{N}} = 4E_{CS}(\mathcal{N}_S - 1/2)$, we have for $E_{JS} < \Gamma$:

$$S_Q(\omega) = \gamma_J(\delta_{\mathcal{N}})\frac{1+\frac{(8\delta_{\mathcal{N}}\omega)^2}{E_{JS}^4/2}\gamma_J(\delta_{\mathcal{N}}-\omega)\gamma_J(\delta_{\mathcal{N}}+\omega)}{[4\gamma_J(\delta_{\mathcal{N}}+\omega)\gamma_J(\delta_{\mathcal{N}}-\omega)]+\omega^2} \qquad (6)$$

Measurement Rate– To determine the measurement time $\tau_{\rm meas}$, we extend our density matrix description to also include m, the number of electrons that have tunnelled through the left SSET junction [4,14] [19]. We are thus able to calculate the distribution of tunnelled electrons $P(m,t|i)$, where $i = \uparrow, \downarrow$ denotes the initial state of the qubit. $\tau_{\rm meas}$ is defined as the minimum time needed before the two distributions $P(m,t|\uparrow)$ and $P(m,t|\downarrow)$ are statistically distinguishable [4]:

$$\frac{1}{\tau_{\rm meas}} = \left(\frac{I_\uparrow - I_\downarrow}{\sqrt{2f_\uparrow I_\uparrow} + \sqrt{2f\downarrow I_\downarrow}}\right)^2, \qquad (7)$$

Here, $I_\uparrow$ and $I_\downarrow$ are the average SSET currents associated with the two qubit states, and $f_\uparrow$ and $f_\downarrow$ are the associated Fano factors which govern the zero-frequency shot noise in the current (i.e. the distribution $P(m,t|i)$ is roughly Gaussian with a mean $(I_i/e)t$ and standard deviation $\sqrt{f_i(I_i/e)t}$). In the absence of the qubit, analysis of the density matrix equations for the SSET reveals:

$$f(\delta) = \frac{3}{2}\left[1 - \frac{1}{2}\frac{E_{JS}^2\left(3(\Gamma/2)^2 - \delta^2\right)}{\left([\Gamma/2]^2 + \delta^2 + E_{JS}^2/2\right)^2}\right], \qquad (8)$$

where we take $eV_{DS} = E_{CS}$, $\delta = \delta_{\mathcal{N}} = 4E_{CS}(\mathcal{N}_S - 1/2)$. Eq. (8) indicates that the effective charge of the carriers in the DJQP process is $3e/2$ in the limit where $\Gamma \gg E_{JS}$. In this limit, Cooper-pair tunneling is the rate-limiting step in the cycle; electrons effectively tunnel in clumps of e or $2e$, leading to an average charge of $3e/2$. A similar argument holds in the opposite regime $\Gamma \ll E_{JS}$.

We consider $\tau_{\rm meas}$ in the limit of weak coupling ($E_{\rm int} \ll \Omega$) and weak mixing ($E_{JQ} \ll \Omega$). In this limit, the two qubit states simply provide an effective shift in the SSET gate voltage. Taking $\delta_V = 0$ and $\delta_{\mathcal{N}} = \Gamma/2$ for near optimal gain, and using Eqs. (6-8), we find that the intrinsic signal-to-noise ratio $(\tau_{\rm meas}\Gamma_{\rm mix})^{-1/2}$ of the measurement, in the relevant regime $E_{JS} < \Gamma$, is given by:

$$(S/N)_{DJQP} = \sqrt{\frac{4}{3}}\,|\cot\theta|\,\frac{\Omega}{\Gamma/2}. \qquad (9)$$

Here, $\cot\theta \equiv 4E_{CQ}(1-\mathcal{N}_Q)/E_{JQ}$, and we take $\gamma_J(0) \ll \Omega < E_{CS}$. If a SET in the sequential-tunneling regime is used for the qubit measurement, it was found in Refs. [3,4] that the optimal S/N is given by ($\Omega < E_{CS}$):

$$(S/N)_{SET} = \lambda\,|\cot\theta|\sqrt{\left(\frac{\Omega}{eV_{DS}}\right)^2 + \frac{g^2}{\pi^2}}, \qquad (10)$$

where λ is of order unity. As the quasiparticle transition rate $\Gamma \sim \frac{g}{2\pi}eV_{DS}$, we see that the S/N achieved using DJQP is parametrically larger (in $2\pi/g \gg 1$) than that obtained for the SET. This enhancement results largely from the narrow width of the DJQP feature– the energy scale over which the current changes (and thus the gain) is set by Γ rather than V_{DS}. The gain and S/N ratio of the SET could be improved by working in the cotunneling regime; however, this would result in a much larger τ_{meas} ($\tau_{meas} \propto g^{-2}$), making one more susceptible to unwanted environmental effects. In contrast, the DJQP feature has both a large gain *and* a short τ_{meas} (i.e. $\tau_{meas} \propto 1/g$). Shown in Fig. 3 as a function of $\mathcal{N}_Q$ are $\tau_{\rm meas}, \Gamma_{\rm rel}$ and $\Gamma_{\rm exc}$ for a strongly coupled device ($C_C/C_\Sigma = 0.05$), with all other parameters corresponding to Ref. [13]. We have taken $\delta_V = 0$ and $\delta_{\mathcal{N}} = \Gamma/2$ for optimal gain. Fig. 3 confirms that an excellent measurement is indeed possible, with $(S/N)^2 > 100$. Note that the strong coupling splits the sharp features occurring in the noise (see Fig 2, solid curve).

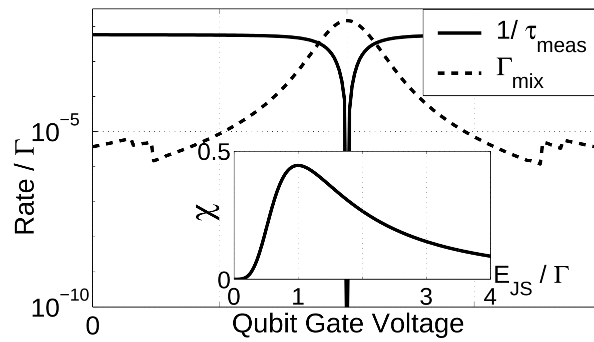

FIG. 3. $1/\tau_{\rm meas}$, $\Gamma_{\rm rel}$, and $\Gamma_{\rm exc}$ vs. qubit gate voltage $\mathcal{N}_Q$ for a strongly coupled system, where $E_{\rm int}/E_{JQ} \simeq 0.3$ (i.e. $C_C/C_\Sigma = 0.05$). A good measurement is possible for a wide range of gate voltages. Inset: Heisenberg efficiency $\chi = \tau_\varphi/\tau_{\rm meas}$ at weak coupling, as a function of E_{JS}/Γ.

We have also studied the "Heisenberg efficiency" $\chi = \tau_\varphi/\tau_{\rm meas}$ of measurement using DJQP for a weak coupling ($E_{\rm int} \ll E_{JS}, \Gamma$) and $\Omega < E_{CS}$, where τ_φ is the measurement-induced dephasing time [18]. Unlike an SET in the sequential tunneling regime, where $\chi \propto g^2$ is always much less than the quantum limit $\chi = 1$ [3,4], here χ is controlled by the ratio E_{JS}/Γ. As shown in the inset of Fig. 3, by tuning this ratio, χ *can be made to approach the quantum limit*. Here, for each value of E_{JS}/Γ we have set V_{DS} and $\mathcal{N}_S$ to optimize the gain. Measurement using DJQP is able to reach a high efficiency when $E_{JS} \simeq \Gamma$ both because of the symmetry of the process, and because of the coherent nature of Josephson tunneling [18]. Clearly, the DJQP process allows for a far superior measurement of a Cooper pair box qubit than a SET.

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[1] R. J. Schoelkopf *et al.*, Science **280**, 1238 (1998).
[2] Korotkov, A. N. *et al.* in *Single-Electron Tunneling and Mesoscopic Physics* (eds. Koch, H. & Lubbig, H.) (Springer, Berlin, 1992).
[3] M. H. Devoret and R. J. Schoelkopf, Nature (London) **406**, 1039 (2000).
[4] Y. Makhlin *et al.*, Phys. Rev. Lett. **85**, 4578 (2000); *ibid.*, Rev. Mod. Phys. **73**, 1039 (2000).
[5] A. Aassime *et al.*, Phys. Rev. Lett. **86**, 3376 (2001).
[6] G. Johansson *et al.*, Phys. Rev. Lett. **88**, 046802 (2002).
[7] T. A. Fulton *et al.*, Phys. Rev. Lett. **63**, 1307 (1989).
[8] D. V. Averin and V. Ya. Aleshkin, JETP Lett. **50**, 367 (1989).
[9] A. Maasen van den Brink *et. al*, **85**, 459 (1991).
[10] Y. Nakamura *et. al*, Phys. Rev. B **53**, 8324 (1996).
[11] S. Pohlen, Ph.D. Thesis, Harvard University (unpublished).
[12] The use of only Cooper-pair tunneling in a modified SSET has been studied in A. B. Zorin, Phys. Rev. Lett. **76**, 4408 (1996); *ibid.*, cond-mat/0112351 (2001). The use

of quasiparticle-only processes in a SSET was studied in A. N. Korotkov, Appl. Phys. Lett. **69**, 2593 (1996).

[13] K. Lehnert *et al.*, submitted to *Science*.

[14] M-S. Choi *et al.*, Phys. Rev. Lett. **87**, 116601-1 (2001).

[15] To observe the population inversion predicted here, relaxation due to the environment must be weak compared to mixing from the measurement. For the devices of Ref. [13], environmental relaxation rates are $\sim 10^6 Hz$; a coupling strength $C_C/C_\Sigma \simeq 0.1$ is then sufficient.

[16] see, e.g., M. Tinkham, *Introduction to Superconductivity*, (McGraw-Hill, New York, 1996).

[17] H. Schoeller and G. Schön, Phys. Rev. B **50**, 18 436 (1994).

[18] A full derivation and discussion will be presented in A. A. Clerk *et al.*, in preparation.

[19] As current is conserved, we could equally as well have chosen the right junction.